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tenus partibus convocatis audias causam et appellatione remota debito fine decidas faciens quod decreveris per censuram ecclesiasticam firmiter observari. Testes autem qui fuerint nominati, si se gratia odio vel timore subtraxerint censura simili appellatione cessante compellas veritati testimonium perhibere. Dat. Viterbii non. Octobr. Pontificatus nostri anno secundo."

A vote of thanks was passed to the Rev. Mr. Thomson and Mr. Brophy for their interesting donations, and to the Rev. Mr. Abbott for his kindness in allowing the Papal Bull to be exhibited to the Academy.

The Rev. Dr. Graves read a paper on a generalization of the Symbolic Statement of Taylor's Theorem.

Lagrange was the first to show that Taylor's theorem admits of the symbolic statement,

$$\phi\,(x+h) = e^{h\frac{d}{dx}}\,\phi(x).$$

The object of the present paper is to discuss the problem of determining that more general symbol which shall have the effect of changing x into any proposed function of x, such as $\psi\,(x)$, throughout the whole of a given function $\phi\,(x)$, which is made the subject of the operation.

The following synthetic course leads readily to the desired end. As the effect of the symbol

$$e^{\frac{d}{dx}}$$

is to change x into $x + 1$, it appears that

$$e^{\frac{d}{df(x)}} \text{ or } e^{\frac{1}{f'(x)}\frac{d}{dx}} \qquad (a)$$

will change $f(x)$ into $f(x) + 1$; and consequently will have the effect of changing x into $\psi\,(x)$ where

$$\psi\,(x) = f^{-1}\{f(x) + 1\} \qquad (b)$$

Developing the symbol of operation (a) we shall now find, as an extension of Taylor's theorem, that

$$\phi\{\psi(x)\} = \phi(x) + \frac{1}{f'(x)}\frac{d\phi(x)}{dx} + \frac{1}{1.2}\frac{1}{f'(x)}\frac{d}{dx}\frac{1}{f'(x)}\frac{d\phi(x)}{dx} + \&c.$$

Here, as in Taylor's theorem, the first term is $\phi\ (x)$; and each of the rest is deduced from the one which precedes it by a uniform process. ψ being given, the form of $f'(x)$, or of its primitive $f(x)$, must be determined by the equation (b). This, as it stands, is a functional equation, but it may be reduced to an equation in finite differences, of the first order, and of a degree which depends on the nature of the function ψ. For, if we make $x = f^{-1}(y)$, it becomes

$$\psi\{f^{-1}(y)\} = f^{-1}(y+1).$$

When, on the other hand, we desire to ascertain the power of any proposed symbol of the form (a), we must first *integrate* $f'(x)$, and then *invert* the function $f\ (x)$, in order to determine the form of ψ. Upon the possibility of effecting these two operations depends the success of this attempt to interpret the symbol. Pursuing this method, we obtain interesting results, of which the following are examples :—

1. The effect of $e^{x\frac{d}{dx}}$ is to change x into ex.

2. The effect of $e^{x^n\frac{d}{dx}}$ is to change x into $\{x^{1-n} + 1 - n\}^{\frac{1}{1-n}}$

3. The effect of $e^{\log m\left\{x + \frac{n}{m-1}\right\}\frac{d}{dx}}$ is to change x into $mx+n$.

4. The effect of $e^{x\log x\frac{d}{dx}}$ is to change x into x^e.

It is worthy of notice, that the general solution of the equation (b) would lead to important results in the theory of functional equations. For we shall have

$$\psi^2(x) = f^{-1}\{f(x) + 2\}$$
$$\psi^3(x) = f^{-1}\{f(x) + 3\}$$
$$\psi^n(x) = f^{-1}\{f(x) + n\}$$

Hence, amongst other consequences, it is evident that any functional equation of the form

$$A_n \psi^n(x) + A_{n-1}, \psi^{n-1}(x) + \ldots . + A \psi(x) + A_0 x = 0,$$

in which A_n, A_{n-1}, &c., are constants, may be reduced at once to a linear equation in finite differences with constant coefficients.

We might also invert the function $\psi(x)$ since

$$\psi^{-1}(x) = f^{-1}\{f(x) - 1\}$$

Dr. Graves stated that, in a continuation of the present paper he would lay before the Academy the results which he had obtained in discussing the symbol

$$e^{L\frac{d}{dx} + M\frac{d}{dy} + N\frac{d}{dz}}$$

in which L, M, N, are functions of x, y, and z; and which has the effect of changing x, y, and z respectively into certain functions of x, y, and z, whose form depends upon that of L, M, and N. One example of this kind has been already communicated to the Academy in a paper read by Dr. Graves on the 9th of June, 1851.

The Chair having been taken by the Rev. Dr. Lloyd,

The President communicated the results of four years' experience, at his own observatory, of the effects produced by the vicinity of a railroad.

"Amid the ever increasing requirements of improved accuracy which the progress of science is pressing on astronomical observers, it becomes important to avoid every possibility of error; to remove every cause that may, in the slightest degree, add to the difficulties that inevitably oppose our advance